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Time-varying Parameters of Inflation Model in Nepal: State Space Modeling

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Abstract

This paper attempts to investigate the stability of time-varying parameters of the random walk model of inflation in Nepal. This study has been motivated with the Lucas Critique (1976) that the monetary/fiscal policy that is exposed to change over time affect the expectations of forward looking economic agents which hence lead to non-constant time-varying parameters of the model. Monthly time series of inflation ranging from August, 1997 to July, 2012 has been utilized for the analysis. Applying the Kalman Filter technique for the estimation of coefficients of random walk model, we found non-constant time varying parameters of both the constant and autoregressive of order one AR(1) coefficient of inflation over the long run. The changes in the expectations of rational economic agents on macroeconomic policies as a result of the problems of policy commitment, credibility and dynamic consistency might have attributed such non-constant time-varying parameters. Therefore, in addition to supply smoothing policies to control inflation in Nepal, consistent and credible policies that are not exposed to change over time may reduce the gap of actual inflation from its targets and hence trigger inflation into desired level.

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CONTENTS

I. INTRODUCTION

The policy credibility, commitment, reputation and time consistency are some of the debated policy concerns both in the developed and developing countries at present. In case of Nepal, the target miss of the macroeconomic variables such as inflation, growth rates of GDP and monetary aggregates and balance of payment surplus raises the issue of policy credibility. Missing the targets of the variables of interest while formulating monetary and fiscal policies creates commitment crisis of the macroeconomic policies and hence efficacy of monetary/fiscal policy. The reason for such crisis may be that the rational economic agents learn about policies commitment and hence change their expectation rules so that it leads to non-constant time-varying economic relationship. Therefore, policy prescription based on constant estimated parameters of those structural and time series model of inflation estimated previously in Nepal may not capture the behavior of time-varying parameters. The issue of parameters that are invariant over time in the modeling came under sever attack with the advent of Lucas Critique (1976). Lucas argues that expectations of economic agents are determined by the policy changes (monetary/fiscal policy rules). The policies that are exposed to change over time lead to parameters of the model to be non-structural.

State-space modeling offers a flexible and encompassing tool to estimate the coefficients of the model that are inherently time-varying making economic relationships potentially unstable. Such a model allows the researcher to model an observed time series as being explained by a vector of (possibly unobserved) state variables which are driven by a stochastic process. Real Business Cycle (RBC) model and Neo-Keynesian models that are framed under Dynamic Stochastic General Equilibrium (DSGE) model are also built under state space framework. These models apply different policy rules in model formulation to address time-varying relationship in the model so that the estimated parameters are considered structural and deep. The Kalman Filter, named after Kalman (1960), is a particular algorithm that is used to solve state space models in the linear case.

Early applications of Kalman Filter to solve state space models in economics include Fama and Gibbons (1982) who model the unobserved ex-ante real interest rate as a state variable that follows an AR(1) process. Clark (1987) uses an unobserved-components model to decompose quarterly real GNP data into the two independent components including a stochastic trend component and a cyclical component. Stock and Watson (1993) used state space model to identify unobserved variables that represents the state of the business cycle. Surveys on the applicability of the state space approach to economics and finance can be found in Hamilton (1994) and Kim and Nelson (1999).

In case of Nepal, various structural models that have examined the inflation behavior include Nepal Rastra Bank (2001), Pant (1988), Khatiwada (1994), Pandey (2005), Nepal Rastra Bank (2007), Bishnoi and Koirala (2006) and (Koirala, 2008b). However, there are limited studies on time varying-parameter of inflation model in different perspectives. An evidence of inflation persistence over different regime shifts in Nepal as found in Koirala's (2012) study confirmed the presence of time-varying parameter of the inflation model. However, the study overlooks the possibility of infinite regimes presented in the economy. A positive relationship between inflation and inflation expectation was found in the study of Koirala (2008a) utilizing adaptive expectation framework. However, adaptive expectation as represented by error learning process relies too much on data while forming expectation. Taylor (1975) and Friedman (1979) have criticized those rational expectations models as they do not address how agents learn the policy that is implemented. Studies on the inflation behavior by capturing different expectation behaviors are the new area of research in Nepal.

In light of limitations in adopting either of the models of adaptive expectation and rational expectation, the time-varying parameters of inflation model utilizing state space model has the advantages of integrating both the expectation theories. Therefore, the objective of this paper is to estimate the time-varying parameters of both the constant and AR(1) parameters of random

walk inflation model utilizing Kalman filter under state space model. In this paper the graphical depiction of both the coefficients of constant and AR(1) coefficient of the inflation model reveals non-constant time-varying parameters depicting the relationship being unstable. Though the estimated parameters rely on economic agents basing all available information and past experiences to make decision about future state of the economy, non-constant parameter over the long run reveals that the economic agents change their decision based on changes in government policies.

II. CONCEPTUAL FRAMEWORK AND METHODOLOGY

Expectations of the economic agents play a central role in determining inflation behavior in an economy. The credibility, commitment, reputation and time consistency of the government policies influence the expectations and hence determine the policy effectiveness in the economy. For instance, if inflation inertia happens due to structural factors (e.g. wage contracts), policy becomes costly and ineffective in reducing inflation. By contrast, if inflation is sticky because of expectations, reducing inflation could be costless in the long run providing that agents change their expectation rule to learn rational expectations equilibrium.

Expectations formation primarily is of two types: adaptive expectation and rational expectation. Either of the theories of expectation have some unsatisfactory assumption so that they do not satisfy Lucas critique (1976). Adaptive expectations are an inadequate concept mostly because they assume that economic agents do not promptly react to systematic mistakes they make (i.e., it relies too much on data). Rational expectations, whereas, have come under attack because they assume too much information on the part of agents and rarely being supported by available data (Fuhrer 1997, Roberts 2001). Rather, the Lucas Critique points out a problem that can occur whenever private agent behavior depends to some degree on government policy rules and this dependence is not taken into account. More precisely the policy rules that are modeled may change over time, and agents learn how these parameters of the model changes over time.

A wide range of linear and nonlinear time series models can be handled with state space modeling. Regression models with changing coefficients, autoregressive integrated moving average (ARIMA) models and unobserved component models can be represented in space state modeling. Estimating HP filter as well as common trends can also be presented in state-space modeling. The distinction between cycles and trends, which lays at the heart of the RBC literature and dynamic general equilibrium models, can be represented with state-space modeling. There are two main types of problems in macroeconomics that can usefully be addressed by using state-space models. Firstly, it can be used to estimate unobservable variables such as potential output. Secondly, such model can effectively handle in estimating time-varying parameters of the model.

The parameters in economic and financial settings change over time so that modeling changes is compelling (Kim and Nelson, 1999). In economics applications we are also regularly confronted with gradually or structurally shifting time series without actually observing the time-varying dynamics. However, it was not until the works of Harvey (1981), Meinhold and Singpurwella (1983) and Hamilton (1988) that applied economists and econometricians began to apply the Kalman filter and the Markov regime switching model. Brown, Durbin and Evans (1975) used CUSUM test to detect instability in the regression coefficients as those coefficients are considered time varying when the cumulative sum of squares moves outside two critical lines. In applied finance, Shanken (1990) took linear functions of observable state variables in measuring time-varying sensitivities.

State space modeling consists of a measurement (observation) equation and a state (transition) equation where the state equation formulates the dynamics of the state variables while the measurement equation relates the observed variables to the unobserved state vector. Let y_t

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denote a $Nx1$ time series of observations whose development over time can be characterized in terms of an unobserved state vector β_t of dimension $Mx1$. Based on this, a standard state space formulation can be represented as follows:

$$y_t = Z_t\beta_t + d_t + \varepsilon_t \qquad\qquad \varepsilon_t \sim N(0, H_t) \qquad\qquad\qquad\qquad (1)$$

$$\beta_t = T_t\beta_{t-1} + c_t + R_t\eta_t \qquad\qquad \eta_t \sim N(0, Q_t) \qquad\qquad\qquad\qquad (2)$$

Equation (1) is the measurement equation, where, y_t is a vector of measured variables of dimension $nx1$, β_t is the state vector of unobserved variables of dimension $px1$, H_t is a matrix of parameters of dimension nxp and $\varepsilon_t \sim N(0, H_t)$. Similarly, equation (2) is the state equation, where F_t is a matrix of parameters and $v_t \sim N(0, Q_t)$. The H_t and Q_t refer to the hyper-parameters of the model. The $mx1$ vector of c_t and $nx1$ vectors of d_t are the deterministic part of state and observation equations respectively. The initial vector of parameter and covariance matrix are assumed ξ_0 and p_0 respectively. The disturbances ε_t and η_t are assumed uncorrelated with each other in all time periods, i.e. $\forall(s,t)\ E(\varepsilon_t, \eta_s') = 0$ and the $\forall t\ E(\varepsilon_t, A_0') = 0$ is uncorrelated with the initial state.

Once a model is put into state space form, the Kalman filter can be used to estimate state vector by filtering. The Kalman filter will provide estimates of the unobserved variable which plays a central role in estimating changes. The purpose of filtering is to update our knowledge of the state vector as soon as a new observation y_t becomes available. Therefore, Kalman filter can be described as an algorithm for the unobserved components at time t based on the available information at the same date. The estimates of any other desired parameters including hyper parameters can be obtained by Maximum Likelihood Estimation (MLE) algorithm as adapted by Shumway and Stoffer (1982). Estimating the states through Kalman filter encompasses three step processes: the initial states, the predict states and the update states.

<u>Initial states</u>

$$\beta_{0\backslash 0},\ P_{0\backslash 0}$$

<u>Predict states</u>

$$\hat{\beta}_{t\backslash t-1} = \mu + F\hat{\beta}_{t-1\backslash t-1} \qquad\qquad\qquad\qquad\qquad \text{............. (1)}$$

$$P_{t\backslash t-1} = F_t P_{t-1\backslash t-1} F_t^T + Q_t \qquad\qquad\qquad\qquad \text{............. (2)}$$

<u>Updates states</u>

$$K_t = P_{t\backslash t-1} H_t^T (H_t P_{t,|t-1} H_t^T + R_t)^{-1} \qquad\qquad \text{............. (3)}$$

$$\hat{\beta}_{t\backslash t} = \hat{\beta}_{t\backslash t-1} + K_t(y_t - H_t\hat{\beta}_{t\backslash t-1}) \qquad\qquad\qquad \text{............. (4)}$$

$$P_{t\backslash t} = (I - K_t H_t)P_{t\backslash t-1} \qquad\qquad\qquad\qquad \text{............. (5)}$$

Where, $\hat{\beta}$ is estimated state, F State transition matrix, P State variance matrix (i.e., error dueto process), y measurement variables, H measurement matrix (i.e., mapping measurements onto state), K Kalman gain, and R measurement variance matrix (i.e, error from measurements). Subscripts $_{t\backslash t}$ current time period, $_{t-1\backslash t-1}$ previous time period, and $_{t\backslash t-1}$ are intermediate steps.

The $\beta_{0\backslash 0}$ and $P_{0\backslash 0}$ are the vectors of initial state and covariance matrix respectively. The covariance matrix $P_{0\backslash 0}$ depicts noise of the $\beta_{0\backslash 0}$. If vector of $\beta_{0\backslash 0}$ and covariance matrix $P_{0\backslash 0}$ are not given prior, $\beta_{0\backslash 0}$ is assumed zero and large number for diagonal elements of matrix $P_{0\backslash 0}$. Equations (1) and (2) are the set of predict equations. Equation (1) is simply the expected value of the transition equation $\mathrm{E}(\mu + F\hat{\beta}_{t-1} + v_t)$ whereas Equation (2) can be described as $VAR(\mu + F\hat{\beta}_{t-1} + v_t)$. Equations (3), (4) and (5) are the set of update equations. The K_t in Equation (3) is termed as Kalman gain which is the weight given to new information. The term in the parenthesis is prediction error. It contains information that is new relative to the previous one. When K_t increases due to uncertainty about state (model noise), it is said to have heavy weight on new information. When K_t falls due to increase in R_t, the shock is said to be less informative. Similarly, Equation (4) updates information of $t-1$ adjusted by K_t which hence is determined by the equation of prediction error $(y_t - H_t\hat{\beta}_{t\backslash t-1})$ in the parenthesis. Equation (5) is the update states of covariance matrix for the state vector.

As the objective of the paper is to estimate and graphically present the time-varying coefficient of random walk model of inflation in Nepal, following model has been specified for the analysis.

$$\pi_t = c_t + b_t \pi_{t-1} + \varepsilon_t, \; VAR(\varepsilon_t) = R \qquad \text{.............. (6)}$$

Assuming π_t an stochastic process (inflation series) generated based on unobserved process of π_{t-1} with c_t and b_t respectively the time-varying coefficients of constant and autoregressive coefficient. Equation (6) can be represented in a state space form as:

$$\begin{pmatrix} c_t \\ b_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} c_{t-1} \\ b_{t-1} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix}, \; VAR\begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix} = Q \qquad \text{.............. (7)}$$

$$\beta_t \qquad \mu \qquad F \qquad \beta_{t-1}$$

Note: $F = 1, \; \pi = 0$

The computation of time-varying parameters in this paper has been programmed in Matlab 2008a. The folder 'Functions' that contains necessary programs for the estimation of Kalman filter is added to Matlab program. The optimization of the model follows Sim-optimization rule that contains four pdf. files applicable for the estimation and are available from Sim's webpage-free download files. The Matlab codes of Kalman filter have been presented in the appendices and are available with the request to author.

III. ESTIMATION RESULTS

The analysis in this paper is based on monthly inflation series starting from August, 1997 to July, 2012. The selected data range sufficiently captures one complete cycle formed on the basis of macroeconomic policies as well as structural and socio-political situation of the country. The Chart 1 depicts the trend of the monthly inflation series over the study period. The increasing trend of inflation during the transition period might have resulted not only from the lack of smooth supply of goods and services but also the inflation expectation of the economic agents

on different policy changes. After the end of the political transition, inflation started to decline for few months and reverted back to previous level of double digit.

Time varying parameters of the inflation model in Nepal is analyzed on the basis of the specification of random walk model as presented in equation (6). Both the constant and AR(1) coefficients of random walk inflation model are examined whether they are changing over time. Non-constant parameters over time may arise as a result of commitment, credibility and dynamic inconsistency problem of macroeconomic policies. Analyzing time-varying parameter of inflation model as the prime objective of this paper, Kalman filter, a particular algorithm, is used to solve state space models. Such filter is used in linear case only.

Chart 1: Inflation Rate
(1997:08 to 2012:12)
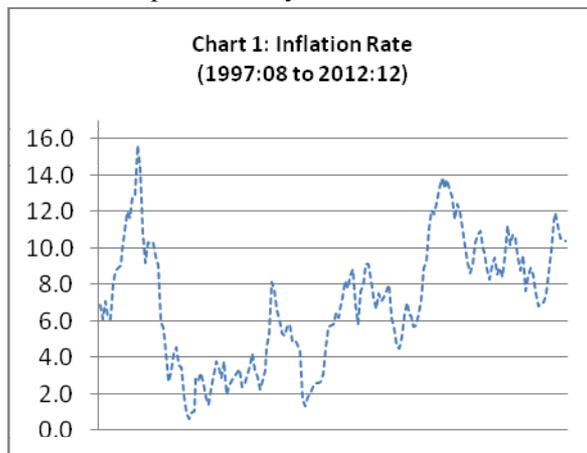
As against the existing time series and structural model of inflation in Nepal, this analysis tries to capture time-varying parameter of inflation model by using Kalman filter. Here parameters are not estimated using regression and maximum likelihood methods, rather they are derived as filter process using latest information available in the form of observable data. We initialize zero for the vector of coefficients as it is not a prior. Further the diagonal elements of the variance the coefficients have been set 100, i.e. $Q = \begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}$ as those coefficients are assumed completely unknown. The selection of diagonal elements of the variance matrix is based on the assumption that we are very unsure about the coefficient estimated and let the data to specify the correct variance of the model. Actually, if we initialize more meaningful values of the variance, we get faster convergence.


The diagonal elements of Q represent system noise whereas R represents measurement noise of the system. The maximum likelihood values of $VAR(\varepsilon_t) = R$ and $VAR\begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix} = Q$ are found to be $R = 0.7$ and $Q = \begin{pmatrix} 0.008 & 0 \\ 0 & 0.002 \end{pmatrix}$ respectively in this analysis. Given the values of R and Q, and the assumption of zero vector of μ and identity matrix of F, the graphical depiction of estimated parameters for the time-varying constant and AR(1) coefficient are presented in Chart 2 and 3 respectively. The respective parameters are the Kalman filter coefficients that are obtained based on the Matlab codes presented in Appendix I. As the result shows that both the parameters of constants and AR(1) coefficients possess convergence. The monotonic divergence of constant and wide fluctuation of AR(1) coefficient

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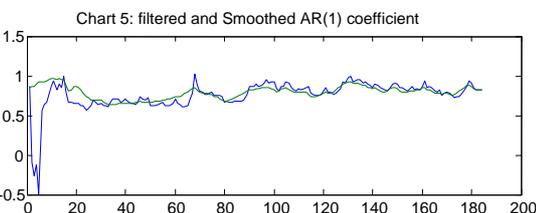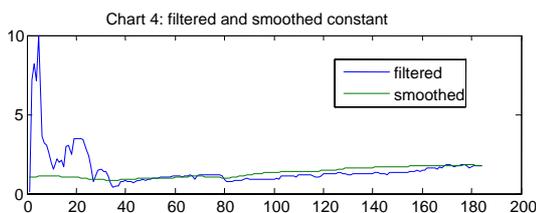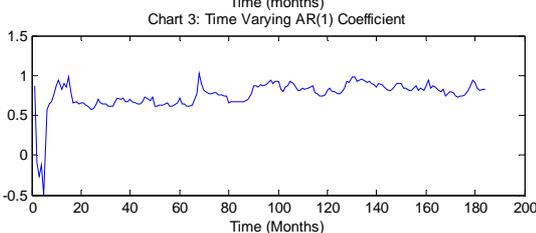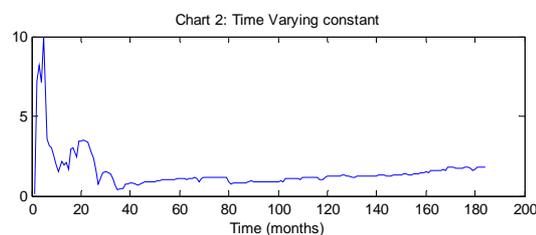
followed by initial convergence over the study period represents lack of constant time-varying coefficients of the model. The AR(1) coefficient is being fluctuated within a certain band. As those coefficients are the coefficient of random walk model of inflation, the estimated value of the variables can be obtained by substituting the estimated coefficients in the random walk model. The Kalman filtered coefficients, as the filter utilizes the latest information while formulating successive filtered values. In case of the use of the model other than the model used in this analysis, time varying parameters of inflation during the initial period have been over estimated whereas such parameters would have been under estimated at the latter periods.

Chart 4 shows filtered and smoothed coefficients of constant while Chart 5 includes filtered and smoothed coefficients of AR(1) coefficient. Kalman filter and smoother equations are programmed in Matlab as shown in Appendix II. The figures depict long run behavior of the coefficients. The smoothed AR(1) coefficient varies within the minimum and maximum values of 0.63 and 0.95 with majority of the values lying within the range of 0.80 to 0.90. Charts 6 and 7 illustrate the parameter of smoothed state estimates with 95% confidence bands. Use is made of Chris Sim's minimiser to calculate filtered and smoothed states at ML values using the Matlab code as presented in Appendix III.

Chart 6: constant

Chart 7: AR(1) coefficient
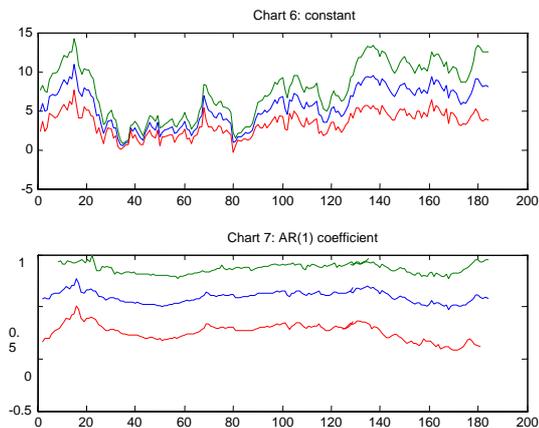
As Kalman filter method has built in specification for updating the estimation of coefficients based on latest available information, it is assumed that expectations of economic agents are captured in estimating coefficients of the model. Therefore, those coefficients are considered consistent coefficient. As both the coefficients are found time-varying over the study period, the prediction of the inflation utilizing those coefficients lead to time varying prediction. As against the constant estimated coefficient by using ordinary least square or maximum likelihood method, the estimated coefficient obtained here are time varying coefficient.

IV. CONCLUSION AND RECOMMENDATION

This paper attempts to investigate whether the time-varying parameters of the random walk model of inflation in Nepal are stable over time. In essence, the unstable time-varying parameters lead to unstable prediction of the variables of interest. Monthly time series of inflation starting from August, 1997 to July, 2012 have been utilized for the analysis. Applying the Kalman Filter technique for the estimation of the parameters of random walk model with the help of Matlab codes as presented in Appendices, we found monotonic divergence of constant and wide fluctuation of AR(1) coefficient followed by initial convergence of both the parameters of random walk model, which represents lack of constant time-varying parameters. Therefore, the findings here in this paper do not validate the presumption of stable random walk model of inflation (the stable relationship of current inflation to the one period lagged inflation) as investigated previously in Nepal. One of the pertinent reasons for such time-varying parameters might be the changes in the expectations of rational economic agents on macroeconomic policies as a result of commit, credibility and dynamic consistency problems.

Therefore, the target miss of inflation over the past several years in Nepal might have resulted from commitment crisis of the macroeconomic policies and hence reducing efficacy of those policies. Therefore, in addition to supply smoothing policies to control inflation, consistent and credible policies that are not exposed to change over time may reduce the gap of actual inflation from its targets and hence trigger inflation into desired level.

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Appendix I: Kalman Filter Codes

```matlab
addpath('functions')
%type in name of data file
Y = xlsread('data/data.xls');
T=rows(Y);
X=[ones(T,1) lag0(Y,1) ];
%remove missing obs
Y=Y(2:T);
X=X(2:T,:);
T=rows(X);
%Step 1 Set Initial State for
the Kalman filter
b00=[0 0]; %initial state
p00=eye(2,2)*100; %initial
covariance
%Step 2 Set up matrices for
the state space
F=eye(2,2);
MU=zeros(1,2);
Q=diag([0.006;0.001]);
R=0.6;
%kalman filter
beta_tt=zeros(T,2); %will hold
filtered state
ptt=zeros(T,2,2); %will hold
its covariance
beta11=b00;
p11=p00;
for i=1:T
    x=X(i,:);
%Prediction equations
beta10=MU+beta11*F';
p10=F*p11*F'+Q;
yhat=(x*(beta10)')';
eta=Y(i,:)-yhat;
feta=(x*p10*x')+R;
%updating equations
K=(p10*x')*inv(feta);
beta11=(beta10'+K*eta')';
p11=p10-K*(x*p10);
ptt(i,:,:)=p11;
beta_tt(i,:)=beta11;
end
subplot(2,1,1);
plot(beta_tt(:,1));
title('constant')
subplot(2,1,2);
plot(beta_tt(:,2));
title('AR(1) coefficient')
```

Appendix II: Kalman Filter and Smoother Codes

```matlab
addpath('functions')
%type in name of data file
Y = xlsread('data/data.xls');
T=rows(Y);
X=[ones(T,1) lag0(Y,1) ];
%remove missing obs
Y=Y(2:T);
X=X(2:T,:);
T=rows(X);
%Step 1 Set Initial State for the Kalman filter
b00=[0 0]; %initial state
p00=eye(2,2)*100; %initial covariance
%Step 2 Set up matrices for the state space
F=eye(2,2);
MU=zeros(1,2);
Q=diag([0.006;0.001]);
R=0.5929;
%kalman filter
beta_tt=zeros(T,2); %will hold filtered state
ptt=zeros(T,2,2); %will hold its covariance
beta11=b00;
p11=p00;
for i=1:T
    x=X(i,:);
%Prediction equations
beta10=MU+beta11*F';
p10=F*p11*F'+Q;
yhat=(x*(beta10)')';
eta=Y(i,:)-yhat;
feta=(x*p10*x')+R;
%updating equations
K=(p10*x')*inv(feta);
beta11=(beta10'+K*eta')';
p11=p10-K*(x*p10);
ptt(i,:,:)=p11;
beta_tt(i,:)=beta11;
end
%kalman smoother
 beta_TT=zeros(T,2); %will hold smoothed state
pTT=zeros(T,2,2);    %will hold covariance of smoothed
state
i=T;   %time period T
 beta_TT(i,:)=beta_tt(i,:); %smoothed state for T
 pTT(i,:,:)=ptt(i,:,:); %Its covariance for T
 for i=T-1:-1:1  %go backward in time
     pnext=F*squeeze(ptt(i,:,:))*F'+Q;
     ipnext=pinv(pnext);
     pcurrent=squeeze(ptt(i,:,:));
     pTTcurrent=squeeze(pTT(i+1,:,:));
beta12=beta_tt(i,:)+(pcurrent*F'*ipnext*(beta_TT(i+1,
:)-beta_tt(i,:)*F'-MU)')';
     p12=pcurrent+pcurrent*F'*ipnext*(pTTcurrent-
pnext)*ipnext'*F*pcurrent';
     beta_TT(i,:)=beta12;
     pTT(i,:,:)=p12;
 end
se=zeros(T,2);
for i=1:T
    temp=squeeze(pTT(i,:,:));
    se(i,1:2)=sqrt(diag(temp))';
end
h1=figure(1)
subplot(2,1,1);
plot([beta_tt(:,1) beta_TT(:,1)]);
title('constant')
subplot(2,1,2);
plot([beta_tt(:,2) beta_TT(:,2)]);
title('AR(1) coefficient')
legend('filtered','smoothed')
```

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Appendix III: <u>Calculate filtered and smoothed states at ML values</u>

```matlab
addpath('functions')

addpath('sims_Optimization')

%===================================Load Data===============================

Y = xlsread('data/data2.xls'); %type in name of data file

T=rows(Y);

X=[ones(T,1) lag0(Y,1) ];

%remove missing obs

Y=Y(2:T);

X=X(2:T,:);

T=rows(X);

%define starting values

theta0=[-2,-2,-2];

%use Chris Sim's minimiser
 [FF,AA,gh,hess,itct,fcount,retcodeh] =
csminwel('kalmanfilterlik',theta0,eye(3)*.1,[],1e-15,1000,Y,X);

 clc

 se=sqrt(diag(hess));

 tratio=AA'./se;

 disp('------------------')

 disp('Parameters T-ratios');

 disp([AA' tratio]);

 disp('-------------------');

 %calculate filtered and smoothed states at ML values

  [lik,beta_tt,ptt ] = kalmanfilterlik(AA,Y,X );

  [bsmooth,se]=ksmooth(AA,beta_tt,ptt);

 %plot

subplot(2,1,1);

plot([bsmooth(:,1) bsmooth(:,1)+2.*se(:,1) bsmooth(:,1)-2.*se(:,1)] );

title('constant')

subplot(2,1,2);

plot([bsmooth(:,2) bsmooth(:,2)+2.*se(:,2) bsmooth(:,2)-2.*se(:,2)] );

title('AR(1) coefficient')
```